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Letter to the Editor

Comments on "A Direct Numerical Solution to a One-Dimensional Helmholtz Equation"

In a recent issue of this Journal, O'Brien [1] presented a direct numerical method for solving a one-dimensional Helmholtz equation of the form $y_{xx} + f(x)\, y = g(x)$ in some bounded domain. The use of a recursive algorithm inferred in his Eq. (10) instead of Eq. (9) in the actual computation of the unknowns does represent a significant improvement in computing efficiency, for his Eq. (10) can be written as

$$\begin{aligned} w_0 &= 0, \quad & w_j &= w_{j-1} + (-1)^j D_{j-1} g_j , & j &= 1(1)n, \\ z_n &= 0, \quad & z_j &= z_{j+1} + (-1)^{j+1} D_{n-(j+1)} g_{j+1} , & j &= n-1(1)1, \\ & & y_j &= (-1)^j (D_{n-j} w_j + D_{j-1} z_j)/D_n , & j &= 1(1)n. \end{aligned}$$

It can easily be seen that, apart from the determination of the sign, only three additions and five multiplications are required for each unknown. If his Eq. (9) had been used, $(n-1)$ additions and n multiplications would have been needed per unknown for a system of n unknowns. Even with the use of his Eq. (10), however, the method he presented is less efficient than a Gaussian elimination method widely discussed in texts in numerical analysis, (e.g., Wachspress [2]). For the case of constant coefficient with uniform grid spacing treated by O'Brien, this latter scheme would demand only three additions and three multiplications for each unknown. If many sets of three-term linear systems having the same coefficient matrix are to be solved repeatedly, one of the parameters in the recursion algorithm can be computed once and for all, reducing the operational counts to two additions and two multiplications per unknown. Thus while the scheme involved is no more complicated than the one presented by O'Brien, it is about twice as efficient.

As far as accuracy is concerned, for the case of nonpositive f, both schemes are stable with respect to the growth of round off errors. For the case of positive f, even if the existence of a unique solution is insured by requiring the coefficient matrix A to be nonsingular in the manner noted by O'Brien, all is not well computationally even for large n, as his Fig. 1 seems to indicate; for the accuracy of the solution will still depend critically on how well-conditioned A is, and not so much on how large n is. For instance, for $n = 5000$, $\alpha = 3.1$ $(D_n \neq 0)$, according to his Fig. 1, the error in his example would be on the order of 10^{-7}.

However, actual computations using his scheme would give a maximum error with a magnitude of about 10^2, using an IBM 7094, single precision (8-place accuracy); and about 10^{-5}, double precision (16-place accuracy). Thus one must exercise extreme care in the application of schemes such as these for solution of wave equations.

REFERENCES

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2. E. L. WACHSPRESS, "Iterative Solution of Elliptic Systems," p. 20, Prentice-Hall, Englewood Cliffs, N.J., 1966.

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